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SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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SHORT TRICKS
Mathematics
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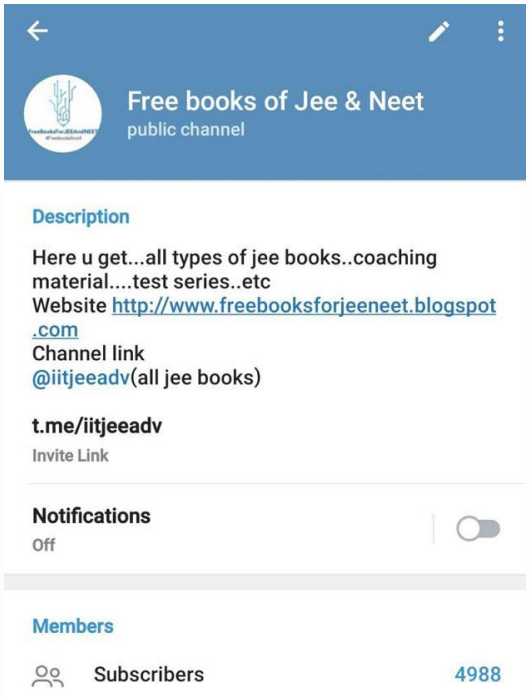

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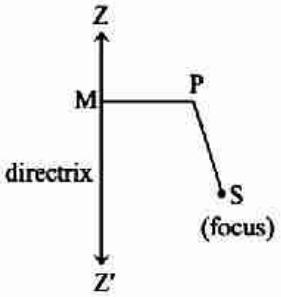
Conic Sections

KEY CONCEPTS

Parabola

A conic is the locus of a point whose distance from a fixed point bears a constant ratio to its distance from a fixed line. The fixed point is the focus S and the fixed line is directrix.



The constant ratio is called "eccentricity" and denoted by "e".

$\frac{PS}{PM} = e$

(i) If e = 1, conic section is parabola

(ii) e > 1, conic section is Hyperbola

(iii) e < 1, conic section is Ellipse

A_1A_2 = Latus rectum = 4 (OS) = 2(SS')

1. General Equation of Conic

A second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents.

(i) Pair of straight lines, if

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(ii) Circle, if a = b, h =0

(iii) Parabola, if $h^2 = ab$ and $\Delta \neq 0$

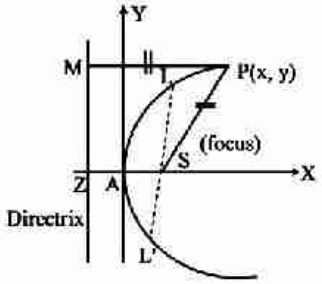
(iv) Ellipse, if $h^2 < ab$ and $\Delta \neq 0$

(v) Hyperbola, if $h^2 > ab$ and $\Delta \neq 0$

(vi) Rectangular hyperbola, if a + b = 0 and $\Delta \neq 0$

2. Parabola

A parabola is the locus of a point which moves in such a way that its distance from a fixed point (focus) is equal to its perpendicular distance from a fixed straight line (directrix).



3. Terms Related to Parabola

(i) Axis : A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola.

(ii) Vertex : The point of intersection of a parabola and its axis is called the vertex of the Parabola.

Note :

➢ The vertex is the middle point of the focus and the point of intersection of axis and directrix.

(iii) Focal Length (Focal distance) :
The distance of any point P (x, y) on the parabola from the focus is called the focal length.

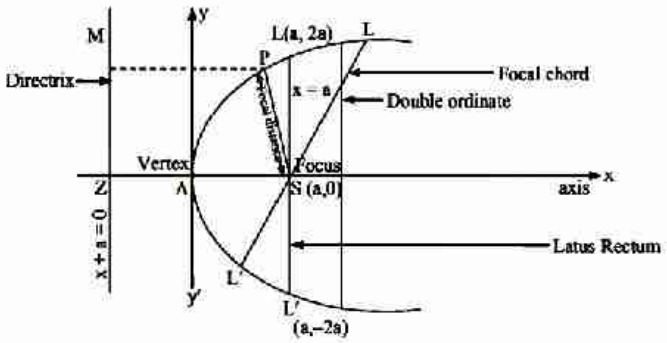
(iv) Double ordinate : The chord which is perpendicular to the axis of Parabola or parallel to Directrix is called double ordinate of the Parabola.

(v) Focal chord : Any chord of the parabola passing through the focus is called Focal chord.

(vi) Latus Rectum : If a double ordinate passes through the focus of parabola then it is called as latus rectum.

4. Standard Equation of Parabola

If we take vertex as the origin, axis as x-axis and distance between vertex and focus as 'a' then equation of the parabola in the simplest form will be. $y^2 = 4ax$



(i) Parameters of the Parabola $y^2 = 4ax$

(a) Vertex A $\Rightarrow$ (0, 0)

(b) Focus S $\Rightarrow$ (a, 0)

(c) Directrix $\Rightarrow x + a = 0$

(d) Axis $\Rightarrow$ y = 0 or x– axis

(e) Equation of Latus Rectum $\Rightarrow$ x = a

(f) Length of L.R. $\Rightarrow$ 4a

(g) Ends of L.R. $\Rightarrow$ (a, 2a), (a, – 2a)

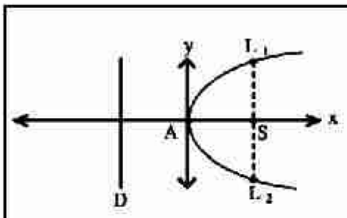
(h) The focal distance $\Rightarrow$ sum of abscissa of the point and distance between vertex and L.R. = $|x_1 + a|$

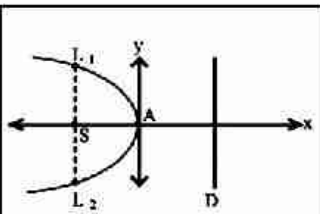
(i) If length of any double ordinate of parabola $y^2 = 4ax$ is 2ℓ then coordinates of end points of this Double ordinate are $\left(\frac{\ell^2}{4a}, \ell\right)$ and $\left(\frac{\ell^2}{4a}, -\ell\right)$.

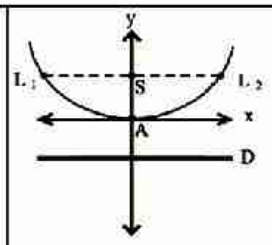
Note :

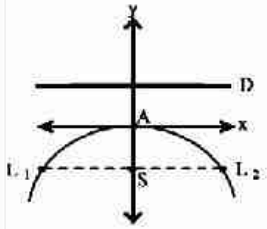
➢ The length of the Latus Rectum = 2 × perpendicular distance of focus from the directrix.

(ii) Other Standard Parabola :









Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of Vertex (A)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Axis of Parabola	y = 0	y = 0	x = 0	x = 0
Coordinates of focus (S)	(a, 0)	(–a, 0)	(0, a)	(0, –a)
Equation of directrix (D)	x = –a	x = a	y = – a	y = a
Length of latus rectum L_1L_2	4a	4a	4a	4a
Coordinates of of L_1, L_2	(a, 2a), (a, –2a)	(–a, 2a), (–a, –2a)	(–2a, a), (2a, a)	(–2a, –a), (2a, – a)

5. Parametric equation of Parabola

The parameter equation parabola $y^2 = 4ax$ are $x = at^2, y = 2at$ Parametric Equation for parabola

(i) $y^2 = -4ax$ is $(-at^2, 2at)$

(ii) $x^2 = 4ay$ is $(2at, at^2)$

(iii) $x^2 = -4ay$ is $(2at, -at^2)$

6. Equation of Chord Joining any two Point of a Parabola

Let $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ be any two point on the parabola $y^2 = 4ax$. Then the equation of the chord joining these points is given by $y(t_1 + t_2) = 2x + 2at_1t_2$

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Important Points to be Remembered

(1) If t_1 & t_2 are the ends of a focal chord of the parabola $y^2 = 4ax$ then $t_1t_2 = -1$, Hence the co-ordinates at the extremities of a focal chord can be taken as $(at^2, 2at)$ & $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$.

(2) Length of the chord $= a(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4}$.

(3) The length of the focal chord joining parameter t_1 and t_2 on the parabola $y^2 = 4ax$ is $a(t_2 - t_1)^2$

(4) If ℓ_1 & ℓ_2 are the lengths of segments of a focal chord of a parabola. Then length of its latus rectum is $\frac{4\ell_1\ell_2}{\ell_1 + \ell_2}$.

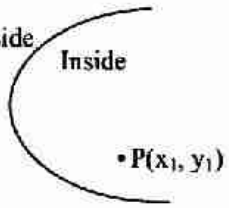
7. Position of a point relative to a parabola

The point (x_1, y_1) lies outside, on or inside the parabola $y^2 = 4ax$ according as the expression $y_1^2 - 4ax_1$ is positive, zero or negative.

$S_1 : y_1^2 - 4ax_1$

$S_1 < 0 \rightarrow$ Inside

$S_1 > 0 \rightarrow$ Outside



8. Line & a Parabola

(i) The line $y = mx + c$ meets the parabola $y^2 = 4ax$ in two points real, coincident or imaginary according as $c <, =$ or $> a/m$

$\Rightarrow$ condition of tangency is $\boxed{c = \frac{a}{m}}$

(ii) Line $y = mx + c$ will be tangent to the parabola $x^2 = 4axy$ if $\boxed{c = -am^2}$

(iii) Then the length of the chord intercepted by the parabola $y^2 = 4ax$ on the line $y = mx + c$ is given by $\boxed{\frac{4}{m^2}\sqrt{a(1+m^2)(a - mc)}}$

(iv) Length of the focal chord making an angle α with the x-axis is $= 4a\,\text{cosec}^2\alpha$.

9. Equation of Tangent

(i) Point Form : The equation of tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1) is $yy_1 = 2a(x + x_1)$ or $T = 0$

(ii) Parametric Form : The equation of the tangent to the parabola at t.

i.e. $(at^2, 2at)$ is $ty = x + at^2$

Note :

➢ The point of intersection of tangents at t_1 and t_2 on the parabola is $[at_1t_2\ ,\ a(t_1 + t_2)]$ i.e. (GM of abscissa, AM of ordinate).

(iii) Slope Form :

(a) The equation of the tangent of the parabola $y^2 = 4ax$ is

$y = mx + \frac{a}{m}$ and the point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

(b) $y = mx - am^2$ is a tangent to the parabola $x^2 = 4ay$ for all value of m and its point of contact is $(2am, am^2)$

(c) The equation of the tangent of slope m to the parabola $(y-k)^2 = 4a(x-h)$ is given by $(y-k) = m(x-h) + \frac{a}{m}$ and the coordinates of point of contact are $\left(h + \frac{a}{m^2}, k + \frac{2a}{m}\right)$

10. Equation of Normal

(i) Point Form : The equation to the normal at the point (x_1, y_1) of the parabola $y^2 = 4ax$ is given by $y - y_1 = \frac{-y_1}{2a}(x - x_1)$

(ii) Slope Form : The equation of normal to the parabola $y^2 = 4ax$ having slope m is $y = mx - 2am - am^3$ and its point of contact is given by $(am^2, -2am)$

Note :

➢ The equation of normal of slope m to $y^2 = -4ax$, $x^2 = 4ay$ and $x^2 = -4ay$ are $y = mx + 2am + am^3$, $x = \frac{y}{m} - \frac{2a}{m} - \frac{a^3}{m}$ and $x = \frac{y}{m} + \frac{2a}{m} + \frac{a^3}{m}$ respectively.

(iii) Parametric Form : The equation to the normal at the point $(at^2, 2at)$ is $y + tx = 2at + at^3$

Important Points to be Remembered

(1) If the normals to the parabola $y^2 = 4ax$ at the point t_1 meets the parabola again at the point t_2 then $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

(2) If the normals to the parabola $y^2 = 4ax$ at the point t_1 and t_2 intersect again on the parabola at point t_3 then $t_1t_2 = 2$ & $t_3 = -(t_1 + t_2)$

(3) The point of intersection of normals at t_1 & t_2 are $(a(t_1^2 + t_2^2 + t_1t_2 + 2), -at_1t_2(t_1 + t_2))$

(4) The length of normal chord $= a(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4} = \frac{4a(t_1^2 + 1)^{3/2}}{t_1^2}$

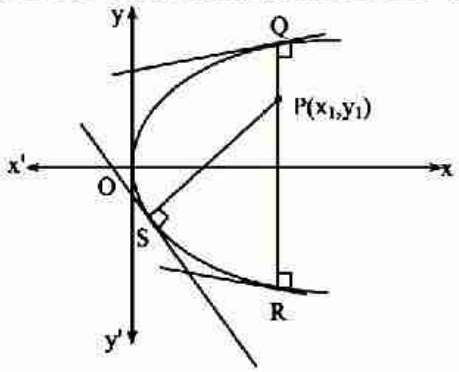
(5) The tangent at one extremity of the focal chord of parabola is parallel to the normal of other extremity.

(6) The normal at any point of a parabola is equally inclined to the focal distance of the point and the axis of the parabola.

(7) Three normals can be drawn from a point to a parabola.

11. Co-Normal Point

The points on the curve at which the normals pass through a common point are called co-normal points. Q, R, S are co-normal points. The co-normal points are also called the feet of the normal.



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Note :

- The sum of the slopes of the normals at co-normals points is zero.
  $m_1 + m_2 + m_3 = 0$
- The sum of the ordinate of the co-normal points is zero.
- The centroid of the triangle formed by the co-normal points lies on the axis of the parabola.

12. Pair of Tangents

The equation to the pair of tangents which can be drawn from any point (x_1, y_1) to the parabola $y^2 = 4ax$ is given by $SS_1 = T^2$ where $S = y^2 - 4ax$; $S_1 = y_1^2 - 4ax_1$; $T = yy_1 - 2a(x + x_1)$

13. Chord of Contact

(i) The equation of chord of contact of tangents drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ is $T = 0$ or $yy_1 = 2a(x + x_1)$.

(ii) Lengths of the chord of contact is $\frac{1}{a}\sqrt{(y_1^2 - 4ax)(y_1^2 + 4a^2)}$

(iii) Area of triangle formed by tangents drawn from (x_1, y_1) and their chord of contact is $\frac{1}{2a}(y_1^2 - 4ax_1)^{3/2}$

14. Director Circle

The locus of the point of intersection of the perpendicular tangents to the parabola $y^2 = 4ax$ is called the director circle. Its equation is $x + a = 0$ which is parabola own directrix.

Note :

- In case of parabola director circle is a line.

15. Equation of the chord with a given middle point

Equation of the chord of the parabola $y^2 = 4ax$ whose middle point is (x_1, y_1) will be given by

$(y - y_1) = \frac{2a}{y_1}(x - x_1)$

This can be reduced to $T = S_1$

where $T = yy_1 - 2a(x + x_1)$ & $S_1 = y_1^2 - 4ax_1$

16. Pole and Polar

(i) Equation of polar of the point (x_1, y_1) with respect to parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$ or $T = 0$

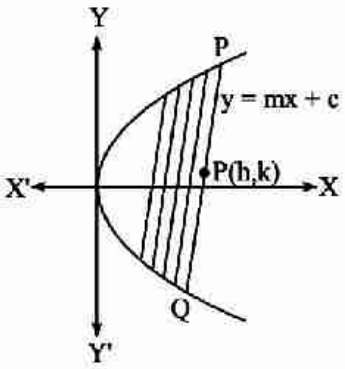
(ii) The pole of the line $\ell x + my + n = 0$ with respect to the parabola $y^2 = 4ax$ is $\left(\frac{n}{\ell}, \frac{-2am}{\ell}\right)$

(iii) If the polar of a point P passes through the point Q, then the polar of Q goes through P, then P and Q are called **conjugate points**.

(iv) Two straight lines are said to be conjugated to each other w.r.t. a parabola when the pole of one lies on the other.

17. Diameter of the Parabola

The locus of the mid points of a system of parallel chords of a parabola is known as diameter of a parabola.



The equation of the diameter bisecting chords of slope m of the parabola $y^2 = 4ax$ is $y = 2a/m$.

18. Geometrical properties of the Parabola

(i) The semi latus rectum of a parabola is the H.M. between the segments of any focal chord of a parabola i.e. if PQR is a focal chord, then $2a = \frac{2PQ.QR}{PQ+QR}$

(ii) The tangents at the extremities of any focal chord of a parabola intersect at right angles and their point of intersection lies on directrix i.e. the locus of the point of intersection of perpendicular tangents is directrix.

(iii) If the tangent and normal at any point P of parabola meet the axes in T and G respectively, then
(a) ST = SG = SP
(b) $\angle$ PSK is a right angle, where K is the point where the tangent at P meets the directrix.
(c) The tangent at P is equally inclined to the axis and the focal distance.

(iv) The area of triangle formed inside the parabola $y^2 = 4ax$ is
$\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ where y_1, y_2, y_3 are ordinate of vertices of the triangle.

(v) The abscissa of point of intersection R of tangents at $P(x_1, y_1)$ and $Q(x_2, y_2)$ on the parabola is G.M. of abscissa of P and Q and ordinate of R is A.M. of ordinate of P and Q thus $R\left(\sqrt{x_1x_2}, \frac{x_1+x_2}{2}\right)$

(vi) The area of triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

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Ellipse

It is a locus of a point which moves in such a way that the ratio of its distance from a fixed point and a fixed line (not passes through fixed point and all points and line lies in same plane) is constant (e) which is less than one.

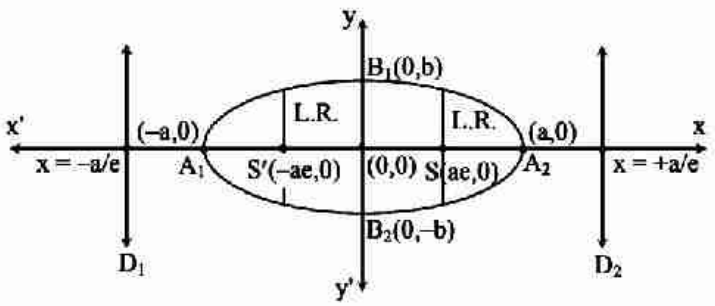
OR

In other word, an ellipse is the locus of a point which moves in a plane so that the sum of it distances from two fixed points is constant.

1. Standard Form of the Equation of Ellipse

(i) Horizontal Ellipse :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (when $a > b > 0$)



(a) Centre (0, 0)

(b) Length of major axis $A_1A_2 = 2a$

(c) Length of minor axis $B_1B_2 = 2b$

(d) Vertices (a, 0) and (–a, 0)

(e) Directrices : $x = \frac{a}{e}$ and $x = -\frac{a}{e}$

(f) Foci : S (ae, 0) & S' (– ae, 0)

(g) Length of Latus Rectum $= \frac{2b^2}{a}$.

(h) Ends of Latus Rectum : $\left(ae, \pm\frac{b^2}{a}\right)$ and $\left(-ae, \pm\frac{b^2}{a}\right)$

(i) Eccentricity $b^2 = a^2(1 - e^2) \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}}$

(j) Focal distance :

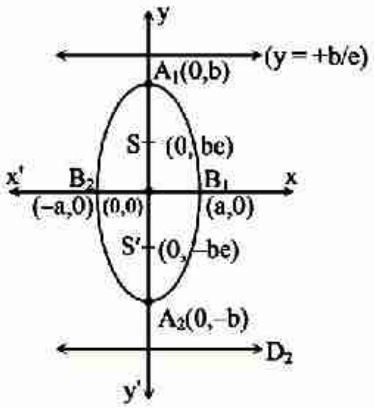
$SP = a - ex_1$ or $S'P = a + ex_1 \Rightarrow SP + S'P = 2a$ (major axis)

(k) Distance between foci = 2ae

(l) Disatnce between directrices = $\frac{2a}{e}$

(ii) Vertical Ellipse :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (when $0 < b < a$)



(a) Centre (0, 0)

(b) Length of Major axis $A_1A_2 = 2b$

(c) Length of Minor axis $B_1B_2 = 2a$

(d) Vertices (0, b) and (0, –b)

(e) Directrices, $y = \pm \frac{b}{e}$,

(f) Foci (0, ± be)

(g) Length of latus rectum = $\frac{2a^2}{b}$

(h) Eccentricity $a^2 = b^2 (1 - e^2) \Rightarrow e = \sqrt{1 - \frac{a^2}{b^2}}$

(i) Focal distance :

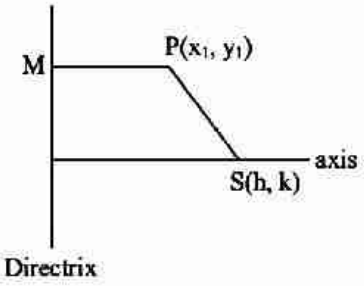
$SP = b - ey_1$ or $S'P = b + ey_1 \Rightarrow SP + S'P = 2b$ (major axis)

(j) Distance between foci = 2be

(k) Distance between directrices = $\frac{2b}{e}$

2. General Equation of an Ellipse

The general equation of an ellipse whose focus is S(h, k) and the directrix is the line $ax + by + c = 0$ and the eccentricity will be e is.



$SP = e\,PM$

$$\Rightarrow (x_1 - h)^2 + (y_1 - k)^2 = \frac{e^2(ax_1 + by_1 + c)^2}{a^2 + b^2}$$

Hence the locus of (x_1, y_1) will be given by

$(a^2 + b^2)[(x - h)^2 + (y - k)^2] = e^2 (ax + by + c)^2$

Note :

➢ *Condition for second degree in X & Y to represent an ellipse is that if* $h^2 - ab < 0$ *&* $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$.

3. Special form of Ellipse

If centre of the ellipse is (h, k) and the direction of the axes are parallel to the coordinate axes, then its equation is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

4. Parametric Form of the Ellipse

Let the equation of ellipse in standard form will be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then the equation of ellipse in the parametric form will be given by $x = a\cos\phi$, $y = b\sin\phi$ where ϕ is the eccentric angle whose value vary from $0 \le \phi < 2\pi$.

5. Position of a point with respect to an Ellipse

The position of a point $P(x_1, y_1)$ with respect to ellipse

$S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ can be given in following manner if

$S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$ then

$S_1 < 0$ Point is inside

$S_1 = 0$ Point lies on ellipse

$S_1 > 0$ Point lies out side the ellipse

6. Line and Ellipse

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two real points, coincident or imaginary according as c^2 is $<$ = or $>$ $a^2m^2 + b^2$

Hence $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

if $c = \pm\sqrt{a^2m^2 + b^2}$

7. Equation of the Tangent

(i) Point form : The equation of the tangent at any point (x_1, y_1) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$. $(T = 0)$

(ii) Parametric form : The equation of tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at any point $(a\cos\phi, b\sin\phi)$ is $\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$.

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(iii) Slope Form : The equation of the tangent of slope of m to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $y = mx \pm \sqrt{a^2m^2+b^2}$ and Its point of contact is $\left(\frac{\mp a^2m}{\sqrt{a^2m^2+b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2+b^2}}\right)$

8. Equation of the Normal

(i) Point form : The equation of the normal at any point (x_1, y_1) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

(ii) Parametric form : The equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at any point $(a\cos\phi, b\sin\phi)$ is

$ax\sec\phi - by\,\text{cosec}\,\phi = a^2 - b^2$

(iii) Slope Form : The equation of the normal of slope m to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are given by $y = mx \pm \frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$ and the coordinate of the foot of normal are $\left(\pm\frac{a^2}{\sqrt{a^2+b^2m^2}}, \pm\frac{b^2m}{\sqrt{a^2+b^2m^2}}\right)$

9. Equation of chord whose mid point is given

The equation of the chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose mid point be (x_1, y_1) is

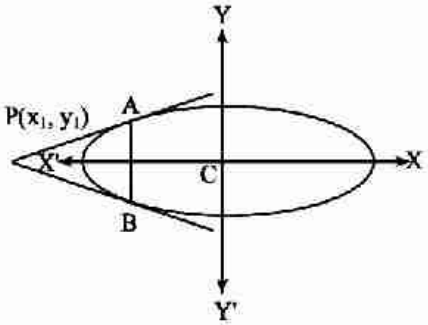
$T = S_1$

where $T = \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$

$S_1 = \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

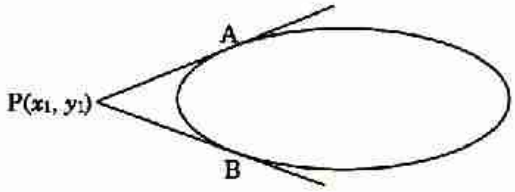
10. Chord of contact

If PA and PB be the tangents through point $P(x_1, y_1)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the equation of the chord of contact AB is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ or $T = 0$



11. Pair of Tangents

Let P (x_1, y_1) be any point lies outside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and let a pair of tangents PA, PB can be drawn to it from P.s



then the equation of pair of tangents of PA & PB is $SS_1 = T^2$ where $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

$S_1 \equiv \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$

$T \equiv \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0.$

12. Pole and Polar

(i) Equation of polar of $P(x_1, y_1)$ w.r.t. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ $(T = 0)$

(ii) The pole of the line $\ell x + my + n = 0$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\left(-\frac{a^2\ell}{n}, -\frac{b^2m}{n}\right)$

13. Director Circle

Locus of the point of intersection of the tangents which meet at right angles is called the **Director Circle.**

The equation of the director circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 + b^2$.

14. Diameter

The locus of the middle points of a system of parallel chords is called a diameter.

If $y = mx + c$ represent a system of parallel chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then the equation of the diameter is $y = -\frac{b^2}{a^2m}x$

15. Conjugate Diameters

Two diameters are said to be conjugate when each bisects all chords parallel to the other.

If m_1 and m_2 be the slope of the conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\boxed{m_1m_2 = \frac{-b^2}{a^2}}$

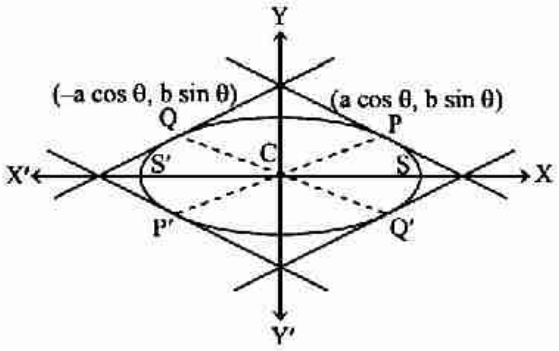
Important Points to be Remembered

(1) The eccentric angles of the extremities of a pair of conjugate diameters differ by a right angle.

(2) The sum of the squares of two conjugate semi diameters of an ellipse is constant and equal to sum of the squares of the semi- axes of the ellipse. i.e. $CP^2 + CD^2 = a^2 + b^2$.

(3) The tangent at the extremities of a pair of conjugate diameters form a parallelogram whose area is constant and equal to product of the axes.

(4) If CP, CQ are two conjugate semi-diameters of an ellipse and S, S′ be two foci of an ellipse, then $SP \times S'P = CQ^2$



16. Auxiliary Circle

A circle described on major axis of ellipse as diameter is called the **auxiliary circle.**

The equation of Auxiliary circle of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$x^2 + y^2 = a^2$ (a > b) or $x^2 + y^2 = b^2$ (b > a)

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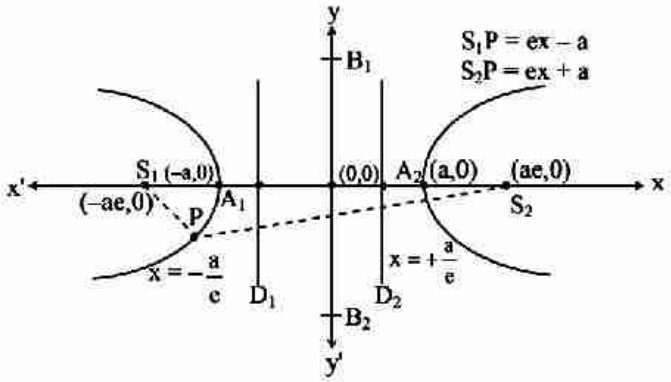
Hyperbola

1. Definition

A hyperbola is defined as the locus of a point moving in a plane in such a way that the ratio of its distance from a fixed point to that from a fixed line (the point does not lie on the line) is a fixed constant greater than 1.

2. Standard Equation of Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



(i) Centre (0, 0)

(ii) Length of transverse axis = 2a

(iii) Length of conjugate axis = 2b

(iv) Vertices (a, 0) and (–a, 0)

(v) Directrices : $x = a/e$ and $x = -a/e$

(vi) Foci : S (ae, 0) and S' (– ae, 0)

(vii) Distance between foci = 2ae

(viii) Distance between directrices = $\frac{2a}{e}$

(ix) Length of Latus Rectum = $\frac{2b^2}{a}$.

(x) End of Latus Rectum $\left(ae, \pm\frac{b^2}{a}\right)$ and $\left(-ae, \pm\frac{b^2}{a}\right)$

(xi) Eccentricity : $b^2 = a^2(e^2 - 1)$ and $e = \sqrt{1 + \frac{b^2}{a^2}}$

(xii) Focal distance : $PS = ex_1 - a$ and $PS' = ex_1 + a$

Note :

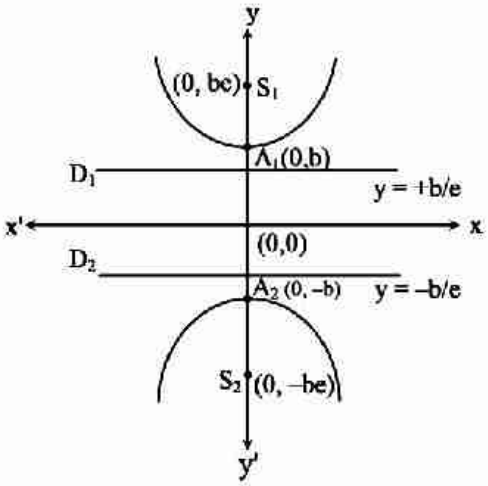
➢ Focal Property : The difference of the focal distances of any point on the hyperbola is constant and equal to transverse axis i.e. $|PS| - |PS'| = 2a$. The distance SS' - focal length.

3. Conjugate Hyperbola

Two hyperbolas such that transverse & conjugate axes of one hyperbola are respectively the conjugate & the transverse axes of the other are called Conjugate Hyperbola of each other.

The equation of the conjugate hyperbola of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$



(i) Centre (0, 0)

(ii) Length of transverse axis = 2b

(iii) Length of conjugate axis = 2a

(iv) Vertices (0, b) and (0, –b)

(v) Directrices : y = b/e and y = – b/e

(vi) Foci : S (0, be) and S' (0, – be)

(vii) Distance between foci = 2be

(viii) Distance between directrices = $\frac{2b}{e}$

(ix) Length of Latus Rectum = $\frac{2a^2}{b}$.

(x) Eccentricity : $b^2 = a^2 (e^2 - 1)$ and $e = \sqrt{1+\frac{b^2}{a^2}}$

Important Points to be Remembered

(1) If e_1 & e_2 are the eccentricities of the hyperbola & its conjugate hyperbola then $\boxed{\frac{1}{e_1^2}+\frac{1}{e_2^2}=1}$.

(2) The foci of a hyperbola and its conjugate hyperbola are concyclic.

(3) Two hyperbolas are said to be similar if they have the same eccentricity.

(4) Two similar hyperbolas are said to be equal if they have same latus rectum.

(5) The equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ will represent an hyperbola if $h^2 - ab > 0$ & $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$.

4. Special Form of Hyperbola

If centre of the hyperbola is (h, k) and the direction of the axes are parallel to the coordinate axes, then its equation is

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

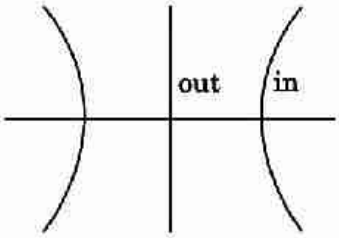
5. Parametric Equation

(i) The parametric equation of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ will be given by $x = a\sec\phi$, $y = b\tan\phi$. (where ϕ is eccentric angle) $(0 \le \phi < 2\pi)$

(ii) The equation $x = a\left(\frac{e^{\theta}+e^{-\theta}}{2}\right)$, $y = b\left(\frac{e^{\theta}-e^{-\theta}}{2}\right)$ are also the parametric equations of the hyperbola.

6. Position of a point with respect to Hyperbola

Let $S = 0$ be the hyperbola and $P(x_1, y_1)$ be the point and $S_1 \equiv S(x_1, y_1)$. Then



$S_1 < 0 \Rightarrow$ P is outside the hyperbola (same side of centre)

$S_1 > 0 \Rightarrow$ P is inside the hyperbola (Opposite side of centre)

$S_1 = 0 \Rightarrow$ P lies on the hyperbola

7. Line and Hyperbola

The straight line $y = mx + c$ is a secant, a tangent or passes outside the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ according as : $c^2 >=< a^2m^2 - b^2$.

Hence the line $y = mx + c$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if

$$\boxed{c=\pm\sqrt{a^2m^2-b^2}}$$

8. Equation of Tangent

(i) **Point form :** The equation of the tangent at any point (x_1, y_1) on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1.\ (T=0)$$

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(ii) **Parametric Form :** Equation of tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(a\sec\theta, b\tan\theta)$ is $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$.

(iii) **Slope form :** The equation of tangent of slope m to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are given by $y=mx\pm\sqrt{a^2m^2-b^2}$ and the point of contacts is $\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}},\pm\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$

9. Equation of the Normal

(i) **Point form :** The equation of normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at (x_1, y_1) is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2=a^2e^2$.

(ii) **Parametric Form :** The equation of normal at $(a\sec\theta, b\tan\theta)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $ax\cos\theta+by\cot\theta=a^2+b^2$.

(iii) **Slope form :** The equation of normal of slope m to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $y=mx-\frac{m(a^2+b^2)}{\sqrt{a^2-b^2m^2}}$ and foot of normal are $\left(\pm\frac{a^2}{\sqrt{a^2-b^2m^2}},\mp\frac{b^2m}{\sqrt{a^2-b^2m^2}}\right)$

10. Pair of Tangents

The equation to the pair of tangents which can be drawn from any point (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is given by : $SS_1=T^2$ where $S=\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; $S_1=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$; $T=\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$

11. Chord of Contact

Chord of contact of tangents drawn from a point outside to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$T=0$ i.e. $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

12. Equation of the chord whose middle point is given

Equation of the chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ whose middle point is (x_1, y_1) is $T=S_1$, where $S_1=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$; $T=\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$.

13. Director Circle

The locus of the point of intersection of perpendicular to tangent to the hyperbola is called a director circle.

The equation of the director circle of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2-b^2$ $(a^2>b^2)$

Note :

➢ If $b^2 > a^2$, then the radius of the director circle is imaginary, so that there is no such circle and so no pair of tangents at right angle can be drawn to the curve.

14. Diameter

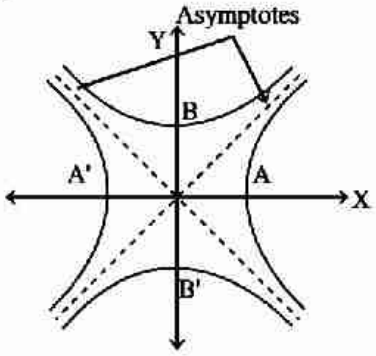
If $y = mx + c$ represent a system of parallel chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then the equation of the diameter is $y = \frac{b^2}{a^2 m}x$.

15. Conjugate Diameter

The diameters of a hyperbola are said to be conjugate diameter, if each bisect the chords parallel to the other. The diameters $y = m_1x$ and m_2x are conjugate, if $m_1m_2 = \frac{b^2}{a^2}$

16. Asymptotes

If the length of perpendicular let fall from a point on a hyperbola straight line ends to zero as the point on the hyperbola moves to the Hyperbola.



Equation of the asymptotes of hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = \pm\frac{b}{a}x$.

Important Points to be Remembered

(1) A hyperbola and its conjugate hyperbola have the same asymptotes.

(2) The angle between the asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\tan^{-1}\left(\frac{b}{a}\right)$

(3) The equation of the pair of Asymptotes differ the hyperbola and the conjugate hyperbola same by the constant only i.e. Hyperbola – Asymptotes = Asymptotes – Conjugate hyperbola.

$H\left(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\right)$, $C\left(\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1\right)$ & $A\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = 0$

Clearly → $\boxed{C + H = 2A}$

(4) The Asymptotes pass through the center of the hyperbola.

17. Rectangular Hyperbola

A hyperbola whose asymptotes include a right angle is said to be rectangular hyperbola or we can say that, if the lengths of transverse and conjugate axes of any hyperbola be equal, then it is said to be a rectangular hyperbola.

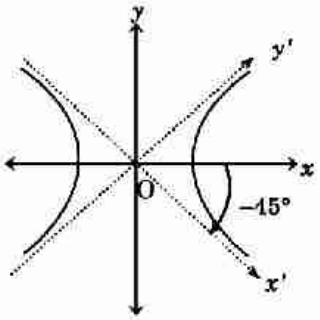
i.e. In a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if b = a, then it said to be rectangular hyperbola. The eccentricity of a rectangular hyperbola is always $\sqrt{2}$.

(i) Rectangular Hyperbola of the Form $x^2 - y^2 = a^2$

(a) Asymptotes are perpendicular lines i.e. $x \pm y = 0$

(b) Eccentricity $e = \sqrt{2}$

(c) Centre (0, 0)



(d) Foci : $(\pm\sqrt{2}\,a, 0)$

(e) Vertices : A(a, 0) and $A_1(-a, 0)$

(f) Directrices : $x = \pm \frac{a}{\sqrt{2}}$

(g) Latusrectum $= 2a$

(h) Parametric form : $x = a\sec\theta$, $y = a\tan\theta$

(i) Equation of tangent : $x\sec\theta - y\tan\theta = a$

(j) Equation of normal : $\frac{x}{\sec\theta} + \frac{y}{\tan\theta} = 2a$

(ii) Rectangular Hyperbola of the form $xy = c^2$

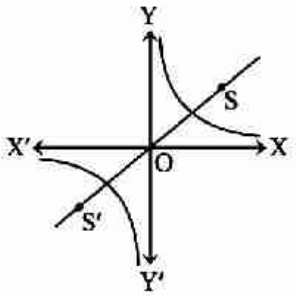
(a) Asymptotes are perpendicular liens i.e. x = 0 and y = 0

(b) Eccentricity : $e = \sqrt{2}$

(c) Centre : (0, 0)

(d) Foci : $S(\sqrt{2}\,c, \sqrt{2}\,c)$, $S_1(-\sqrt{2}\,c, -\sqrt{2}\,c)$

(e) Vertices : A(c, c), $A_1(-c, -c)$



(f) Directrices : $x + y = \pm \sqrt{2}\, e$

(g) Latusrectum $= 2\sqrt{2}\, c$

(h) Parametric form : $x = ct,\ y = \frac{c}{t}$.

18. Equation of Tangent of Rectangular Hyperbola $xy=c^2$

(i) Point Form : The equation of tangent at x_1, y_1 to the rectangular hyperbola is $xy_1 + yx_1 = 2c^2$ or $\frac{x}{x_1} + \frac{y}{y_1} = 2$

(ii) Parametric Form : The equation of tangent at $\left(ct, \frac{c}{t}\right)$ to the rectangular hyperbola is $x + yt^2 - 2ct = 0$

19. Equation of Normal of Rectangular Hyperbola $xy=c^2$

(i) Point Form : The equation of the normal at (x_1, y_1) to the rectangular hyperbola is $xx_1 - yy_1 = x_1^2 - y_1^2$

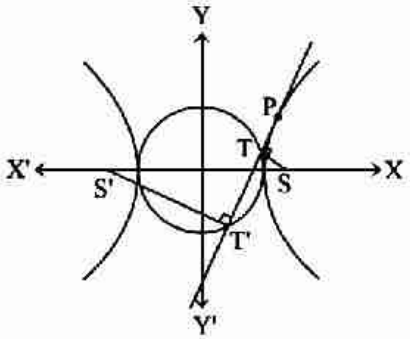
(ii) Parametric Form : The equation of Normal at $\left(ct, \frac{c}{t}\right)$ to the rectangular hyperbola is $xt^3 - yt - ct^4 + c = 0$

(iii) Four normals can be drawn from a point to the hyperbola $xy = c^2$.

20. Equation of Auxiliary Circle

Locus of the feet of the perpendicular drawn from focus of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on any tangent is its auxiliary circle

Equation of auxilliary circle is $x^2 + y^2 = a^2$



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SHORT TRICKS

Trick -1 Length of latus rectum of parabola

(i) Length of latus rectum for the parabola $ax^2 + by + cx + d = 0$ is $= \left|\frac{\text{coeff. of } y}{\text{coeff. of } x^2}\right|$

(ii) Length of latus rectum for the parabola $py^2 + qy + rx + s = 0$ is $= \left|\frac{\text{coeff. of } x}{\text{coeff. of } y^2}\right|$

Q.1 The length of latus rectum of the parabola $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ is-

(A) $\frac{2u^2\cos^2\alpha}{g}$ (B) $\frac{u^2\sin^2 2\alpha}{g}$ (C) $\frac{u^2\cos^2 2\alpha}{g}$ (D) $\frac{u^2\cos^2\alpha}{g}$

Sol. **[A]**

Proper Method

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\frac{gx^2}{2u^2\cos^2\alpha} - x\tan\alpha = -y$$

$$\Rightarrow \left(x^2 - x\frac{2u^2\cos^2\alpha\tan\alpha}{g}\right) = -y\frac{2u^2\cos^2\alpha}{g}$$

$$\Rightarrow \left(x - \frac{u^2\cos^2\alpha\tan\alpha}{g}\right)^2 = \frac{-2u^2\cos^2\alpha}{g}\, y + \frac{u^4\cos^4\alpha\tan^2\alpha}{g^2}$$

$$\Rightarrow \left(x - \frac{u^2\cos^2\alpha\tan\alpha}{g}\right)^2 = -\frac{2u^2\cos^2\alpha}{g}\left(y - \frac{u^2\cos^2\alpha\tan^2\alpha}{2g}\right)$$

$\therefore$ length of L.R $= \frac{2u^2\cos^2\alpha}{g}$

Short Trick

Length of L.R $= \left|\frac{1}{\frac{-g}{2u^2\cos^2\alpha}}\right|$

$= \frac{2u^2\cos^2\alpha}{g}$

Q.2 The length of the latus rectum of the parabola $x = ay^2 + by + c$ is-

(A) $\frac{a}{4}$ (B) $\frac{a}{3}$ (C) $\frac{1}{a}$ (D) $\frac{1}{4a}$

Sol. **[C]**

Proper Method

$x = ay^2 + by + c$

$$\Rightarrow x = a\left[y^2 + \frac{b}{a}y + \frac{c}{a}\right]$$

$$\Rightarrow x = a\left[y^2 + 2.\frac{b}{2a}y + \frac{b^2}{4a^2}\right] + c - \frac{b^2}{4a}$$

$$\Rightarrow \left(y + \frac{b}{2a}\right)^2 = 4\left(\frac{1}{4a}\right)\left(x + \frac{b^2 - 4ac}{4a}\right)$$

$\Rightarrow Y^2 = 4AX$

Length of LR $= 4 \times \frac{1}{4a} = \frac{1}{a}$

Short Trick

Length $= \left|\frac{\text{coeff. of } x}{\text{coeff. of } y^2}\right| = \left|\frac{1}{a}\right|$

Q.3 The length of the latus-rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is- **[AIEEE-2002]**

(A) 1 (B) 6 (C) 8 (D) 10

Sol. **[C]**

Proper Method	**Short Trick**
$x^2 - 4x - 8y + 12 = 0$ $\Rightarrow x^2 - 4x = 8y - 12$ $\Rightarrow (x-2)^2 = 8y - 12 + 4$ $\Rightarrow (x-2)^2 = 8y - 8$ $\Rightarrow (x-2)^2 = 8(y-1)$ $\therefore$ length of L.R = 8	Length = $\left\lvert\frac{\text{coeff. of y}}{\text{coeff. of } x^2}\right\rvert = \left\lvert\frac{-8}{1}\right\rvert = 8$

Q.4 Find the latus-rectum of the parabola $3x^2 - 6x - y + 6 = 0$ is-

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{2}{3}$ (D) $\frac{2}{7}$

Sol. **[A]**

Proper Method	**Short Trick**
$3x^2 - 6x - y + 6 = 0$ $\Rightarrow 3x^2 - 6x = y - 6$ $\Rightarrow x^2 - 2x = \frac{y}{3} - \frac{6}{3}$ $\Rightarrow (x-1)^2 = \frac{y}{3} - 2 + 1$ $\Rightarrow (x-1)^2 = \frac{1}{3}(y-3)$ comparing with $X^2 = 4aY$ $(X = x-1, Y = y-3, a = \frac{1}{12})$ length of L.R = $4a = \frac{1}{3}$	Length = $\left\lvert\frac{\text{coeff. of y}}{\text{coeff. of } x^2}\right\rvert = \left\lvert\frac{-1}{3}\right\rvert = \frac{1}{3}$

Trick -2 If parabola is in the form
$ax^2 + bx + cy + d = 0$ ..... (1)
or $py^2 + qy + rx + s = 0$ ..... (2)
then to find equation of axis
For parabola (1) $\Rightarrow$ differentiate given equation w.r.t. x keeping y as constant
For parabola (2) $\Rightarrow$ differentiate given equation w.r.t. y keeping x as constant

Q.5 The equation of the axis of the parabola $x^2 - 4x - 3y + 10 = 0$ is

(A) $y + 2 = 0$ (B) $x + 2 = 0$ (C) $x - 2 = 0$ (D) $y - 2 = 0$

Sol. **[C]**

Proper Method	**Short Trick**
$x^2 - 4x - 3y + 10 = 0$ $\Rightarrow x^2 - 4x + 4 = 3y - 6$ $\Rightarrow (x-2)^2 = 4.\frac{3}{4}(y-2)$ $(x-2)^2 = 4.\frac{3}{4}(y-2)$ comparing with $X^2 = 4aY$ equation of axis $x - 2 = 0$	Differentiating given equation w.r.t. x keeping y as constant $2x - 4 = 0$ $\Rightarrow x - 2 = 0$

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Q.6 The axis of the parabola $4y^2 - 6x - 4y = 5$ is-

(A) $y - 1 = 0$ (B) $2y + 1 = 0$ (C) $2y - 1 = 0$ (D) $y - 2 = 0$

Sol. **[C]**

Proper Method	**Short Trick**
Here $4y^2 - 4y = 6x + 5$ $\Rightarrow 4(y^2 - y) = 6x + 5$ $\Rightarrow 4\left(y - \frac{1}{2}\right)^2 = 6x + 5 + 1$ $\Rightarrow \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}(6x + 6)$ $\Rightarrow \left(y - \frac{1}{2}\right)^2 = \frac{6}{4}(x + 1)$ Comparing with $Y^2 = 4aX$, then equation of axis is $y - \frac{1}{2} = 0$ $2y - 1 = 0$	Differentiating w.r.t y keeping x as constant $8y - 4 = 0$ $\Rightarrow 2y - 1 = 0$

Q.7 Find the axis of the parabola $3x^2 - 6x - y + 6 = 0$ is-

(A) $x - 2 = 0$ (B) $x - 1 = 0$ (C) $x + 1 = 0$ (D) $2x - 1 = 0$

Sol. **[B]**

Proper Method	**Short Trick**
$3x^2 - 6x - y + 6 = 0$ $\Rightarrow 3x^2 - 6x = y - 6$ $\Rightarrow x^2 - 2x = \frac{y}{3} - \frac{6}{3}$ $\Rightarrow (x - 1)^2 = \frac{y}{3} - 2 + 1$ $\Rightarrow (x - 1)^2 = \frac{1}{3}(y - 3)$ comparing with $X^2 = 4aY$ $(X = x - 1, Y = y - 3, a = \frac{1}{12})$ axis : $X = 0 \Rightarrow x - 1 = 0$	Differentiating w.r.t x keeping y as constant $6x - 6 = 0$ $\Rightarrow x - 1 = 0$

Trick -3 Tangents drawn from point (x_1, y_1) to the parabola $y^2 = 4ax$ makes an angle θ, then

$$\tan\theta = \left|\frac{\sqrt{S_1}}{x_1 + a}\right| \text{ (where } S_1 = y_1^2 - 4ax_1)$$

Q.8 Tangents are drawn from the point $(-2, -1)$ to the parabola $y^2 = 4x$. If α is the angle between these tangents then $\tan\alpha$ equals - **JEE Main Online 2014**

(A) 3 (B) 2 (C) 1/3 (D) 1/2

Sol. **[A]**

Proper Method	**Short Trick**
Any tangent to $y^2 = 4x$ is $y = mx + 1/m$ If it is drawn from $(-2, -1)$, then $-1 = -2m + 1/m$ $\Rightarrow 2m^2 - m - 1 = 0$ If $m = m_1, m_2$ then $m_1 + m_2 = 1/2$, $m_1m_2 = -1/2$ $\therefore \tan a \frac{m_1 - m_2}{1+m_1m_2} = \frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1+m_1m_2}$ $\frac{\sqrt{1/4+2}}{1-1/2} = 3$	$\tan\alpha = \left\lvert\frac{\sqrt{1+8}}{-2+1}\right\rvert = 3$

Trick -4 **Eccentricity of an ellipse**

Eccentricity of an ellipse $ax^2 + by^2 + cx + dy + e = 0$ is

If $a > b$, then $e = \sqrt{1 - \frac{\text{coeff. of } y^2}{\text{coeff. of } x^2}}$

If $b > a$, then $e = \sqrt{1 - \frac{\text{coeff. of } x^2}{\text{coeff. of } y^2}}$

Q.9 The equation $x^2 + 4y^2 + 2x + 16y + 13 = 0$ represents a ellipse then eccentricity is-

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

Sol. **[C]**

Proper Method	**Short Trick**
We have $x^2 + 4y^2 + 2x + 16y + 13 = 0$ $(x^2 + 2x + 1) + 4(y^2 + 4y + 4) = 4$ $(x + 1)^2 + 4(y + 2)^2 = 4$ $\frac{(x+1)^2}{2^2} + \frac{(y+2)^2}{1^2} = 1$ Comparing with $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$ where $X = x + 1, Y = y + 2$ and $a = 2, b = 1$ eccentricity of the ellipse $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$	$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Q.10 The eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is-

(A) 1/3 (B) 2/3 (C) 3/4 (D) 5/9

Sol. [B]

Proper Method	Short Trick
$9x^2 + 5(y^2 - 6y + 9) = 45$ $\Rightarrow \frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$ $e = \sqrt{1 - \frac{a^2}{b^2}}$ $(\because b > a)$ $e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$	$e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

Q.11 Eccentricity of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$ is-

(A) $1/\sqrt{3}$ (B) $\sqrt{3}/2$ (C) 1/2 (D) 1/4

Sol. [B]

Proper Method	Short Trick
$4x^2 + y^2 - 8x + 2y + 1 = 0$ $\Rightarrow 4x^2 - 8x + y^2 + 2y = -1$ $\Rightarrow 4(x^2 - 2x) + y^2 + 2y = -1$ $\Rightarrow 4(x-1)^2 + (y+1)^2 = -1 + 4 + 1$ $\Rightarrow \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$ $e = \sqrt{1 - \frac{a^2}{b^2}}$ $(b > a)$ $e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$	$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Q.12 The eccentricity of the ellipse represented by the equation $25x^2 + 16y^2 - 150x - 175 = 0$ is -

(A) 2/5 (B) 3/5 (C) 4/5 (D) 7/5

Sol. [B]

Proper Method	Short Trick
$25x^2 + 16y^2 - 150x - 175 = 0$ $\Rightarrow 25x^2 - 150x + 16y^2 = 175$ $\Rightarrow 25(x^2 - 6x) + 16y^2 = 175$ $\Rightarrow 25(x-3)^2 + 16y^2 = 175 + 225$ $\Rightarrow \frac{(x-3)^2}{16} + \frac{y^2}{25} = 1$ $e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$	$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Trick -5 Length of latus rectum of an ellipse

Let the equation of ellipse $ax^2 + by^2 + cx + dx + e = 0$

Length of latus rectum $= \frac{2(\text{coff. of } y^2)}{\sqrt{\text{coff. of } x^2}}$ $(a > b)$

Length of latus rectum $= \frac{2(\text{coeff. of } x^2)}{\sqrt{\text{coeff. of } y^2}}$ $(b > a)$

Q.13 Latus rectum of ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$ is-

(A) 8/3 (B) 4/3 (C) $\frac{\sqrt{5}}{3}$ (D) 16/3

Sol. **[A]**

Proper Method	**Short Trick**
$4(x^2 - 2x + 1) + 9(y^2 - 4y + 4) = 36$ $\Rightarrow \frac{4(x-1)^2}{36} + \frac{9(y-2)^2}{36} = 1$ $\Rightarrow \frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ Latus rectum $= \frac{2\times 4}{3} = \frac{8}{3}$	Length of latus rectum $= \frac{2(4)}{\sqrt{9}} = \frac{8}{3}$

Q.14 The length of the latus rectum of ellipse $9x^2 + 16y^2 - 36x + 96y + 36 = 0$ is.

(A) $\frac{9}{2}$ (B) $\frac{7}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{4}$

Sol. **[A]**

Proper Method	**Short Trick**
Given: $9x^2 + 16y^2 - 36x + 96y + 36 = 0$ ...(1) $\Rightarrow [9x^2 - 36x] + [16y^2 + 96y] = -36$ $\Rightarrow 9[x^2 - 4x + 4] + 16[y^2 + 6y + 9] = -36 + 36 + 144$ $\Rightarrow 9[x-2]^2 + 16[y+3]^2 = 144$ Dividing throughout by 144, we get $\frac{[x-2]^2}{16} + \frac{[y+3]^2}{9} = 1$ ...(2) Length of LR $= \frac{2b^2}{a} = \frac{2(9)}{4} = \frac{9}{2}$ units.	length of the latus rectum $= \frac{2(9)}{\sqrt{16}} = \frac{9}{2}$

Trick -6 Eccentricity of hyperbola

Let the equation of hyperbola is $ax^2 - by^2 + cx + dy + e = 0$, then

$$e = \sqrt{1 + \frac{\text{coeff. of } x^2}{|\text{coeff. of } y^2|}} = \sqrt{1 + \frac{a}{b}}$$

Q.15 The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola, then its eccentricity is

(A) $\frac{16}{3}$ (B) $\frac{4}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\sqrt{\frac{19}{3}}$

Sol. **[D]**

Proper Method	**Short Trick**
$16(x^2 - 2x + 1) - 3(y^2 - 4y + 4) = 48$ $\Rightarrow \frac{(x-1)^2}{3} - \frac{(y-2)^2}{16} = 1$ $e = \frac{\sqrt{19}}{\sqrt{3}}$	$e = \sqrt{1 + \frac{16}{3}} = \sqrt{\frac{19}{3}}$

Q.16 The eccentricity of the hyperbola $9x^2 - 4y^2 + 18x - 8y - 31 = 0$ is -

(A) $\sqrt{\frac{11}{2}}$ (B) $\sqrt{\frac{13}{4}}$ (C) $\sqrt{\frac{7}{4}}$ (D) $\sqrt{\frac{9}{2}}$

Sol. **[B]**

Proper Method

Given: $9x^2 - 4y^2 + 18x - 8y - 31 = 0$ ...(1)

$\Rightarrow 9(x^2 + 2x + 1) - 4(y^2 + 2y + 1) = 31 + 9 - 4$

$\Rightarrow 9(x + 1)^2 - 4(y + 1)^2 = 36$

Dividing throughout by 36, we get

$$\frac{(x+1)^2}{4} - \frac{(y+1)^2}{9} = 1 \quad \text{...(2)}$$

Eccentricity

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{13}}{2}$$

Short Trick

$$\sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

Q.17 The eccentricity of $x^2 - 3y^2 - 4x - 6y - 11 = 0$ is -

(A) $\frac{2}{\sqrt{5}}$ (B) $\frac{5}{\sqrt{3}}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{2}}$

Sol. **[C]**

Proper Method

Given : $x^2 - 3y^2 - 4x - 6y - 11 = 0$ ...(1)

$\Rightarrow (x^2 - 4x) - 3(y^2 + 2y) = 11$

$\Rightarrow (x^2 - 4x + 4) - 3(y^2 + 2y + 1) = 11 + 4 - 3$

$\Rightarrow [x - 2]^2 - 3[y + 1]^2 = 12$

$$\Rightarrow \frac{[x-2]^2}{12} - \frac{[y+1]^2}{4} = 1 \quad \text{...(2)}$$

Eccentricity

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{16}}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Short Trick

$$e = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$$

Trick -7 Question solved with the help of diagram

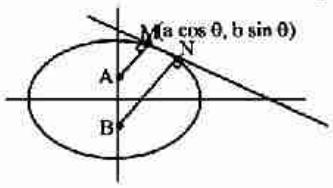
Q.18 The sum of the squares of the perpendicular on any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from two points on the minor axis each distance $\sqrt{a^2 - b^2}$ from the centre is -

(A) a^2 (B) b^2 (C) $2a^2$ (D) $2b^2$

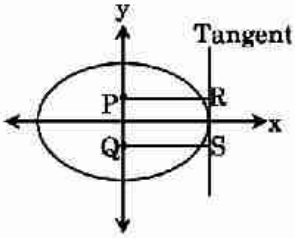
Sol. **[C]**

Proper Method

Equation of tangent is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$



Short Trick



$(PR)^2 + (QS)^2 = a^2 + a^2 = 2a^2$

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$$AM = \frac{0+\frac{\sqrt{a^2-b^2}}{b}\sin\theta-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

$$BN = \frac{0-\frac{\sqrt{a^2-b^2}}{b}\sin\theta-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

$$\text{Now } AM^2+BN^2 = \frac{2\left(\frac{(a^2-b^2)}{b^2}\sin^2\theta+1\right)}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$= \frac{2(a^2\sin^2\theta-b^2\sin^2\theta+b^2)}{b^2}\times\frac{a^2b^2}{(b^2\cos^2\theta+a^2\sin^2\theta)}$$

$$= \frac{2a^2(a^2\sin^2\theta+b^2\cos^2\theta)}{(b^2\cos^2\theta+a^2\sin^2\theta)}$$

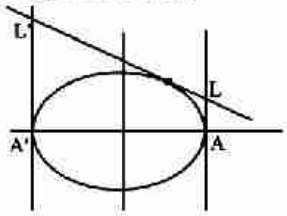
$$= 2a^2$$

Q.19 The tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the tangents at the vertices A, A' in L and L'. Then AL. A'L' =

(A) a + b (B) $a^2 + b^2$ (C) a^2 (D) b^2

Sol. [D]

Proper Method



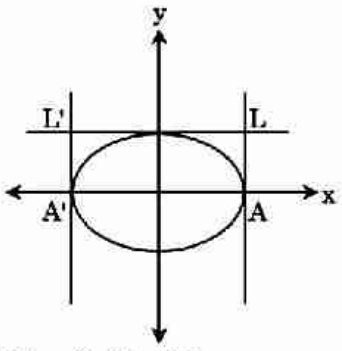
$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

$$L \equiv \left(a, \frac{b(1-\cos\theta)}{\sin\theta}\right)$$

$$L' \equiv \left(-a, \frac{b(1+\cos\theta)}{\sin\theta}\right)$$

$$\text{Now } AL \cdot A'L' = \frac{b^2(1-\cos^2\theta)}{\sin^2\theta} = b^2$$

Short Trick



(AL) (A'L') = (b) (b) = b^2

Q.20 The product of perpendiculars on a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ drawn its foci is

(A) a^2 (B) b^2 (C) $a^2 + b^2$ (D) $\sqrt{a^2+b^2}$

Sol. **[B]**

Proper Method

Equation of any tangent to the given ellipse is

$$\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$$

$$\Rightarrow bx\cos\phi + ay\sin\phi = ab \quad \ldots. (1)$$

Product of perpendiculars drawn from foci (ae, 0), (–ae, 0) on (1)

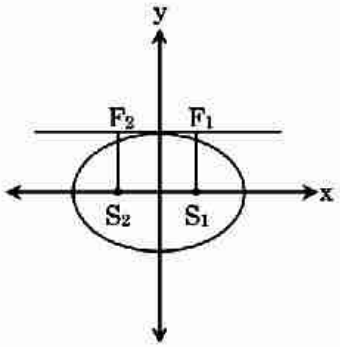
$$= \frac{|(abe\cos\phi - ab)(-abe\cos\phi - ab)|}{b^2\cos^2\phi + a^2\sin^2\phi}$$

$$= \frac{b^2(a^2 - a^2e^2\cos^2\phi)}{b^2\cos^2\phi + a^2\sin^2\phi}$$

$$= \frac{b^2[a^2 - (a^2 - b^2)\cos^2\phi]}{b^2\cos^2\phi + a^2\sin^2\phi} \quad (\because a^2e^2 = a^2 - b^2)$$

$$= \frac{b^2(b^2\cos^2\phi + a^2\sin^2\phi)}{b^2\cos^2\phi + a^2\sin^2\phi} = b^2$$

Short Trick



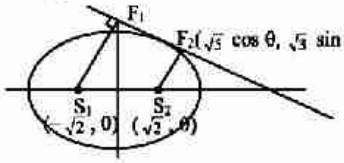
$(S_1F_1)(S_2F_2) = b^2$

Q.21 If F_1 and F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5} + \frac{y^2}{3} = 1$ on the tangent at any point P on the ellipse, then $(S_1F_1).(S_2F_2)$ is equal to-

(A) 2 (B) 3 (C) 4 (D) 5

Sol. **[B]**

Proper Method



$$e = \frac{\sqrt{5-3}}{\sqrt{5}} = \sqrt{\frac{2}{5}}$$

$$ae = \sqrt{2}$$

$$\frac{x\cos\theta}{\sqrt{5}} + \frac{y\sin\theta}{\sqrt{3}} = 1$$

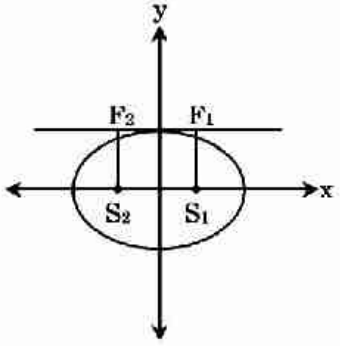
$(S_2F_2)\ (S_1F_1)$

$$= \frac{\left(\frac{-\sqrt{2}\cos\theta}{\sqrt{5}} - 1\right)}{\sqrt{\frac{\cos^2\theta}{5} + \frac{\sin^2\theta}{3}}} \times \frac{\left(\frac{\sqrt{2}\cos\theta}{\sqrt{5}} - 1\right)}{\sqrt{\frac{\cos^2\theta}{5} + \frac{\sin^2\theta}{3}}}$$

$$= -\frac{\left(\frac{2}{5}\cos^2\theta - 1\right)}{\frac{\cos^2\theta}{5} + \frac{1-\cos^2\theta}{3}}$$

$$= \frac{5 - 2\cos^2\theta}{5} \times \frac{15}{3\cos^2\theta + 5 - 5\cos^2\theta} = 3$$

Short Trick



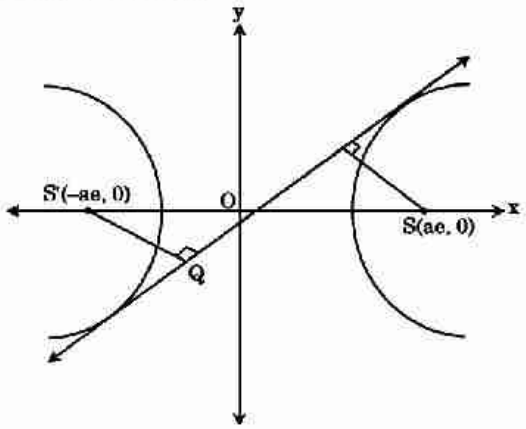
$(S_1F_1)(S_2F_2) = b^2 = 3$

Q.22 The product of the lengths of the perpendiculars drawn from foci on any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is -

(A) a^2 (B) b^2 (C) a^2b^2 (D) a^2 / b^2

Sol. **[B]**

Proper Method



Let the equation of tangent $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

$$SP.S'Q = \left|\frac{\frac{ae}{a}\sec\theta - 1}{\sqrt{\frac{\sec^2\theta}{a^2} + \frac{\tan^2\theta}{b^2}}}\right| \left|\frac{-\frac{ae}{a}\sec\theta - 1}{\sqrt{\frac{\sec^2\theta}{a^2} + \frac{\tan^2\theta}{b^2}}}\right|$$

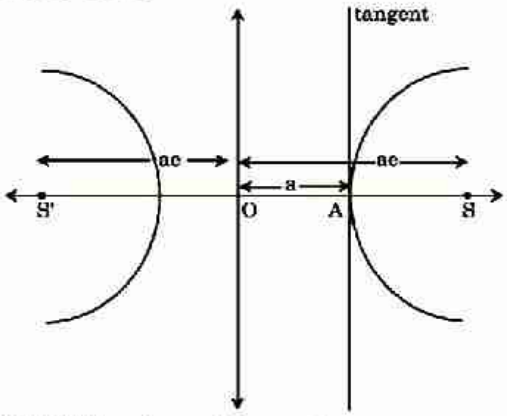
$$SP.S'Q = \frac{e^2\sec^2\theta - 1}{\frac{\sec^2\theta}{a^2} + \frac{\tan^2\theta}{b^2}} = \frac{a^2b^2(e^2\sec^2\theta - 1)}{b^2\sec^2\theta + a^2\tan^2\theta}$$

$$= \frac{a^2b^2(e^2\sec^2\theta - 1)}{a^2(e^2-1)\sec^2\theta + a^2(\sec^2\theta - 1)}$$

$$= \frac{a^2b^2(e^2\sec^2\theta - 1)}{a^2(e^2\sec^2\theta - 1)}$$

$= b^2$

Short Trick



(SA) (S'A) = (ae – a) (ae + a)

$= a^2 (e^2 - 1)$

$= b^2$

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